A GROUP THEORETICAL MATHEMATICAL MODEL OF SHIFTS INTO HIGHER LEVELS OF CONSCIOUSNESS IN KEN WILBER'S INTEGRAL THEORY

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Utilizing Ken Wilber's Integral theory of levels of consciousness as described in a number of his books (see for example "Sex, Ecology, Spirituality", "Integral Psychology", " A Brief History Of Everything", "A Theory Of Everything", etc), we shall apply a basic group theoretical mathematical model to describe how a person may shift into higher levels of consciousness. As Wilber explains, levels of consciousness may vary greatly across various streams; for example, a person may be on a rational level in cognition, a mythic level in morals, and a psychic level in spirituality, etc. Unless stated otherwise we will refer to a generic level of consciousness to mean an overall or general average level of consciousness across all the various streams that Wilber describes in his books. We note that Wilber uses the terminology waves and structures and levels interchangeably, as well as streams and lines interchangeably. We shall also refer to Wilber's use of the term "states", which refer to various temporary modes of consciousness, such as waking, dreaming, altered, meditation, etc. Specifically we will look at a group theoretical mathematical model to describe how a particular state of consciousness may have significant impact on a person shifting into a higher level of consciousness.

We make the assumption that for a person who is on a continuum between two levels of consciousness, such as rational and vision-logic, vision-logic and psychic, psychic and subtle, etc., there are states of consciousness the person may experience which may move him/her toward the higher level of consciousness. For example, if I am in the middle of a continuum between the rational and vision-logic levels of consciousness, prolonged periods of meditation over a certain time period may be a significant factor in enabling me to move closer toward the vision-logic level of consciousness. We shall refer to this type of meditation experience in accordance with Ken Wilber's model of "An Integral Calculus of Indigenous Perspectives", as described in his personal communications on his Integral Institute website. Let us assume that our meditation experience has the format 1p(1p) X 1p(1-p) X 1p(1/p) in Wilber's model, where 1p(1p) refers to the person's awareness of him/herself, 1p(1-p) refers to the person viewing him/herself subjectively, 1p(1/p) refers to the person him/herself being viewed in a terminating perspective of levels of depth, and X is a symbol denoting the product of these three entities (see Appendix B: "An Integral Mathematics of Primordial Perspectives" in Wilber's Integral Calculus of Indigenous Perspectives article for more detailed information of these terms). Since Wilber's mathematical notation is quite cumbersome, we shall more simply refer to the above phenomenon as M mod x , which means that person x is experiencing this meditation M within him/herself to him/herself via him/herself. It is understood that we are focusing upon some particular type of meditation represented by M, and a particular individual represented by X, as our theory is attempting to capture the subjective individualized framework of both the person and the type of mediation experienced. We will utilize a somewhat arbitrary rating scale from

1 to 10 to indicate the strength of impact of continued meditation practice over a certain time period in relation to moving from the rational to vision-logic level of consciousness. We will make the assumption that the lower the rating number, the stronger the impact of the experience; let us give our meditation experience a roughly intermediary rating rank of 6. We now utilize some mathematical group theory, particularly the theory of cyclic groups.

A mathematical group is defined to be a set of elements S with an operation * such that 1) if x is an element of S and y is an element of S then x*y is also an element in S; 2) there is an identity element I in S such that for all elements x in S, x*I = I*x = x; 3) for all elements x, y, and z in S we have x*(y*z) = (x*y)*z (associative law); 4) for each element x in S there exists an element y in S such that x*y = y*x = I; we refer to this element y as x^(-1) and call it the inverse element of x. If for all elements x and y in S we also have the property that x*y = y*x then our group is referred to as a commutative (or abelian) group. An easy example of a commutative group is the infinite set of integers S = {...-3,-2,-1,0,1,2,3,...} under addition. We see that the sum of two integers is always an integer, zero is the identity element, the associative law holds, for any integer x we have x^(-1) = -x, and for any integers x and y we have x + y = y + x; thus the set of integers is a commutative group under addition. We will not overdo our introduction to group theory here, but you may want to think about why the integers are not a group under multiplication, taking 1 to be the identity element. If there is an element x in our group S such that every element y in S can be written as x^n for some integer n, then our group S is referred to as a cyclic group with the generator x. x^n here refers to (x*x*x...*x) n times if n > 0, x^(-n) = (x^(-1))^n, and we define x^0 = I, the identity element of the group S. It is an easy mathematical exercise to prove that all cyclic groups are commutative. We see that our infinite commutative group of integers is actually a cyclic group generated by 1. For an example of a finite commutative cyclic group consisting of 12 elements, think of the hours of an ordinary 12 hour clock; we see that 1 is a generator of the group, 12 is the identity element (12 + 5 = 5, 12 + 7 = 7, 12 + (-4) = 12 + 8 = 8, etc.), and for any hour hand clock number x we have x ^ 12 = (x + x + x + ...) (12 times) = 12 (the symbol ^ denotes that the next number or letter is an exponent).

We now propose a mathematical model to describe shifts into higher levels of consciousness in a logical fashion, utilizing a variation of a group theoretical mathematical structure. To formulate a group theoretical model of shifts into higher levels of consciousness, we shall define the transition into the next higher level of consciousness to be a "biquasi-identity element" of a "biquasi-group" (these "biquasi" terms will soon be defined). Thus in our above meditation example, the transition from the rational to the vision-logic level of consciousness is what we define as the biquasi-identity element I in a cyclic biquasi- group S generated by M mod x. We shall interpret the equation (M mod x)^6 = I to mean that continued practice of our meditation over a certain time period will be a significant factor in enabling person x to move from the rational to the vision-logic level of consciousness. If we think of our 1 to 10 rating scale rank as denoting a context of time, where the higher the number means the greater the amount of time spent in the activity, then we have the building blocks of a mathematical group theoretical model of shifts into higher levels of consciousness. We can think of our meditation example as resembling a finite cyclic group S consisting of 6 elements generated by $M \bmod x$; we say "resembling" as opposed to actual due to the biquasi nature of our group, which we shall now describe. For simplistic illustrative purposes, lets make the assumption that $M \bmod x$ denotes 30 hours of meditation over the time period of one month. According to our group theoretical model, person x would make significant progress toward moving into the vision-logic level of consciousness if he/she were to diligently continue this meditation practice for 180 hours over a 6 month time period, which is the meaning of the equation $(M \bmod x)^6 = I$. We have the equations $M \bmod x + M \bmod x = (M \bmod x)^2$; $(M \bmod x)^2 + (M \bmod x)^3 = (M \bmod x)^5$; in general we can say $(M \bmod x)^m + (M \bmod x)^n = (M \bmod x)^{(m+n)}$ for positive integers m and n. However, once person x reaches the vision-logic level of consciousness, the impact of his/her meditation practice is no longer in the same way relevant to progressing to the next higher (psychic in this case) level of consciousness, as whole other factors may come into the picture. We therefore define $I*I = I$ and $I*(M \bmod x)^n = (M_0 \bmod x)^n$, where $M_0 \bmod x$ means that person x is now meditating while in a vision-logic level of consciousness. Note that by using this scheme, we would have $I*(M \bmod x)^2 = I*I*(M \bmod x)^2 = (M_0 \bmod x)^2$ and $I*(M \bmod x)^{62} = I*(M \bmod x)^2 = (M_0 \bmod x)^2$. We therefore make no further interpretations of the impact of the meditation experience on person x once the vision-logic level of consciousness is reached. Certainly another mathematical scheme could be devised, but at this point we are merely illustrating the essential ideas in its simplest formulation. At any rate, our equation $I*(M \bmod x)^n = (M_0 \bmod x)^n$ resembles the requirements for I to be an identity element of a group, but there are two problems here. The major problem is that M becomes M_0 (signifying we are now in the vision-logic level of consciousness) and a minor problem is that if I is an identity element then $I*(M \bmod x)^n$ must equal $((M \bmod x)^n)*I$. But what exactly does it mean to first meditate $((M \bmod x)^n)$ and then shift into the vision-logic level of consciousness? In order to satisfy the commutative order preserving requirement of an identity element in a group, we make the mathematical definition that $((M \bmod x)^n)*I = I*(M \bmod x)^n = (M_0 \bmod x)^n$. However, the change from M to M_0 does necessitate us calling our identity element something resembling but not quite exactly an identity element; we shall call it a "biquasi-identity element"; the term "biquasi" refers to the fact that we are using a second set to refer to our shift into the vision-logic level of consciousness. In a similar manner, we don't quite have a bona-fide group, but if we use this biquasi-identity element in the required properties of a group, we find that our group properties are essentially satisfied and we now have what we shall refer to as a "biquasi-group"; more specifically we have a "cyclic biquasi-group" generated by $M \bmod x$. (Please see the Appendix at the end of this article entitled "The Mathematical Theory of Biquasi-Groups" for a more formal mathematical definition of a biquasi-group, along with a few basic lemmas that describe some of its properties. There is no relation to our definition of a biquasi-group and the standard definition of a quasigroup in the mathematical literature; the same applies to our definition of “biquasi-identity element” and the following definition of “biquasi-inverse element”.) We note that given an element of the form (M mod x)^n, we have a “biquasi-inverse element” in the analogous way to which we have an inverse element of a group; namely (M mod x)^(6-r) where n = 6z + r in the division algorithm; i.e. z is the largest multiple of 6 that is less than or equal to n. For example, the biquasi-inverse of (M mod x)^20 is (Mod x)^4, since 20 = 6 x 3 + 2, 4 = 6 - 2, and (M mod x)^20 + (M mod x)^4 = (M mod x)^24 = I*I*I*I = I. Of-course the reality in our vision-logic meditation example is that (M mod x)^20 is already in vision-logic and is equal to (M_0 mod x)^2. Another group theoretical fact that is mathematically not at all difficult to prove is the statement that each element in a group has a unique inverse element. This statement remains true for a biquasi-inverse element in a biquasi-group, if we consider M mod x and M_0 mod x = I * M mod x to represent the same inverse element. For example, (M mod x)^20, (M_0 mod x)^2, and (M mod x)^2 are all inverses of (M mod x)^4, and (M mod x)^20 = (M_0 mod x)^2 = I * (M mod x)^2.

Our above mathematical model can describe the relative strength of impact of various states of consciousness upon the levels of consciousness across various streams. For example, lets assume that our above equation (M mod x)^6 = I refers to the transition from the rational to vision-logic level of consciousness in the cognitive stream through our meditation practice. We will therefore refer to our above shift into vision-logic consciousness via meditation equation as ((M mod x)_c)^6 = I_c. If we make the assumption that more prolonged meditation practice is necessary to shift from the rational to the vision-logic level of consciousness in the spirituality stream than in the cognitive stream (note that Wilber actually describes the spirituality stream in terms of a number of streams: care, openness, concern, religious faith, meditative stages, etc.), then perhaps we may have the equation ((M mod x)_s)^9 = I_s. If we attempt to make a mathematical interpretation here, we could say that ((M mod x)_c)^6 = ((M mod x)_s)^9 or (M mod x)_c = ((M mod x)_s)^(1.5). We are dividing both exponents here in the same way that we simplify 6x = 9y to x = 1.5y, as opposed to the usual laws of exponents in algebra; this is because our group notation (M mod x)^6 actually means 6 multiplied by M mod x (180 = 6 x 30 hours of meditation). In other words we would need to meditate one and a half times as long, or 270 hours over a 9 month time period, to make enough significant impact to shift into the vision-logic level of consciousness in the spirituality stream.

In a similar manner, our above mathematical scheme allows us to compare other types of meditative disciplines to our initial meditation experience. For example, lets assume that person x were to engage in some kind of yoga practice under the guidance of a particular yoga master, and that we believe that this yoga practice will result in quicker gains and more impact toward enabling person x to shift from the rational to the vision-logic level of consciousness. In Wilber’s theory of A Calculus of Indigenous Perspectives we may describe this yoga practice under a particular yoga master as 2p(1p) X 2p(1-p) X 1p(1/p), which means that the first person of this particular yoga master is transmitting via his/her first person this yoga experience to the first person of person x. If we refer to our yoga practice as Y mod x to signify 30 hours a month of this type of yoga discipline, our generic vision-logic level of consciousness equation may now look like (Y mod x)^3 = I, meaning that person x will make significant progress in shifting into the vision-logic level of consciousness after 90 hours (3 months) of this disciplined yoga practice. This results in the equation (Y mod x)^3 = (M mod x)^6 or (Y mod x) = (M mod x)^2. We can interpret this to mean that for person x, a yoga practice under a particular yoga master is twice as impactful as a meditation practice; i.e. person x will make the same progress toward the vision-logic level of consciousness in 3 months (90 hours) of instructional yoga practice as opposed to 6 months (180 hours) of meditation practice. We also make the crucial point that the same meditation or yoga discipline may have quite a different impact for different people. In other words, we could have equations like (M mod x)^6 = I and (M mod y)^3 = I, or M mod y = (M mod x)^2, meaning that person x would need to meditate for twice as long (6 months as opposed to 3 months) as person y in order to have the same impact on the transition from the rational to the vision-logic level of consciousness. Of-course all kinds of variations may come into the picture to complicate the mathematics, such as meditating for less time each month over the same 6 month time period, but we will keep things as simple as possible to illustrate our basic mathematical ideas. However, it may be helpful to have a mathematical way of describing when a particular state or activity or experience has no significant impact on enabling a person to shift into a higher level of consciousness. For example, lets now assume that person x starts playing basketball for 30 hours a month in the hopes of shifting his/her consciousness from the rational to the vision-logic level. It seems like a fairly reasonable assumption to say that no matter how long a time period or how many hours person x plays basketball, there will not be any significant impact on person x shifting into the vision-logic level of consciousness (I realize that some basketball enthusiasts may want to debate this statement, but lets assume it is true for now in order to illustrate the mathematics). We shall utilize the notation (B mod x)^(infinity) to mean that even if it were possible for person x to play basketball for an infinite number of hours, this would not have any significant impact on person x moving into the vision-logic level of consciousness. For a brief mathematical aside, taking things to an infinite power may very well yield surprising results; if we take .9 to an infinite power (meaning .9 x .9 x .9 x... an arbitrarily large number of times) we can obtain a number as close to zero as we would like to have, and mathematically we say that the limit of .9 to an arbitrarily large power is zero. Thus we can actually write (.9)^(infinity) = 0 where our operation is now multiplication. This "limit" idea is the mathematical basis of Calculus, and as Ken Wilber states in his introduction to A Calculus of Indigenous Perspectives, his use of the term "Calculus" has nothing to do with the mathematical discipline of Calculus. Indeed, a more accurate title for Wilber's manuscript might be "An Algebra of Indigenous Perspectives".

We may ask what the combined impact is of various meditative disciplines in combination with each other. To take the simplest example of this kind of combination, lets assume that person x now has a

combined practice of 30 hours of meditation and 30 hours of yoga (as previously defined) per month, and that the impact is the same no matter how the meditation and yoga is spaced out in relation to each other (or mathematically our quasi-group is commutative). We refer to this dynamic as (M mod x)*(Y mod x) = (Y mod x)*(M mod x). We can make various assumptions about the combined impact of our new practice, but for illustrative purposes let us assume that in our 1 to 10 rating scale we now have a rating of 2, and therefore the equation ((M mod x)*(Y mod x))^2 = I, meaning that after 120 hours over a 2 month time period, person x will have significant impact to shift into the vision-logic level of consciousness. We still have a cyclic biquasi-group structure, now generated by (M mod x)*(Y mod x), but we also have a second biquasi-group structure consisting of meditation alone, yoga alone, combined meditation and yoga, and the shift into the vision-logic level of consciousness. We can describe this mathematically by the following quasi-group relations: (M mod x)^6 = I, (Y mod x)^3 = I, ((M mod x)*(Y mod x))^2 = I. However, upon further examination we see that our second quasi-group structure is still cyclic, since we have the original equation Y mod x = (M mod x)^2 (meaning that the yoga discipline is twice as impactful as the meditation discipline) and therefore our second biquasi-group is actually generated by M mod x; i.e.
((M mod x)*(Y mod x))^2 = ((M mod x)*((M mod x)^2))^2 = ((Mod x)^3))^2 = (Mod x)^6 = I. In group theoretical language we would say that the biquasi-group generated by (M mod x)^3 is "isomorphic" to the biquasi-group generated by (M mod x)*(Y mod x), meaning that they are generated by elements that have the same impact in regard to person x shifting into the vision-logic level of consciousness (please see the Appendix for the mathematical definition of isomorphic biquasi-groups). However, by changing the nature of our assumptions in our meditation and yoga disciplines, we can formulate a simple example of a non-cyclic biquasi-group with only 4 elements. Let us now assume that our meditation and yoga disciplines are each highly intensified but do not have any beneficial (or harmful) effects from being combined. We shall give each of our disciplines a rating of 2 on our rating scale, meaning that from practicing either meditation or yoga alone, person x will experience significant impact to shift into the vision-logic level of consciousness in 60 hours (2 months of practicing 30 hours of meditation a month or 2 months of practicing 30 hours of yoga a month). But we now assume that this 2 month time period does not change by increasing the practice to a combined 30 hours a month of meditation and 30 hours a month of yoga; this is like saying that increasing the number of hours a month to more than 30 hours a month of meditative discipline (i.e. meditation or instructional yoga) has no significant impact for person x in regard to shifting into the vision-logic level of consciousness. Our equations would therefore become (M mod x)^2 = I, (Y mod x)^2 = I, ((M mod x)*(Y mod x))^2 = I. Notice that we now have a bona-fide non-cyclic quasi-group structure consisting of the four elements M mod x, Y mod x, (M mod x)*(Y mod x), and I (we are assuming that our biquasi group is commutative, i.e. (M mod x)*(Y mod x) = (Y mod x)*(M mod x)). Our biquasi-group is now generated by the two elements M mod x and Y mod x.

Let us take our group theoretical analysis one step further. We define a p-group to be a group with p^n number of elements for some prime number p and positive integer n, and we generalize this definition to define a biquasi p-group to be a biquasi group with p^n number of elements for some prime number p and positive integer n. In our last example, we see that we have a biquasi 2-group consisting of four elements, as we are assuming that our biquasi p-group is commutative, meaning that the order in which we do our meditation and yoga does not matter in regard to shifting into the vision logic level of consciousness. Since (M mod x)*(Y mod x) = (Y mod x)*(M mod x) we have that ((M mod x)*(Y mod x))^2 = ((M mod x)*(Y mod x))*((M mod x)*(Y mod x)) = ((M mod x)*(M mod x))*((Y mod x)*(Y mod x)) = ((M mod x)^2)*((Y mod x)^2) = I. We thus see that it does not change the two month time period required to shift into the vision-logic level of consciousness by doing 30 hours of meditation and 30 hours of yoga per month, and to keep our model as simple as possible we assume that our 30 hours of yoga are not done until our 30 hours of mediation are completed, and vice versa. However, what if it made a difference whether we began our practice with meditation or yoga? Or speaking mathematically, what if (M mod x)*(Y mod x) is not equal to (Y mod x)*(M mod x), meaning that our biquasi 2-group were not commutative? According to mathematical group theory, the smallest 2-group that is not commutative consists of 8 elements, and this remains true for biquasi p-groups; let's examine the possibilities. We first assume that it takes 4 months to shift into the vision-logic level of consciousness by meditation alone, and 2 months for yoga alone; i.e. (M mod x)^4 = I and (Y mod x)(2 = I. If our biquasi-group were commutative, then we would have ((M mod x)*(Y mod x))^2 = (M mod x)*(Y mod x)*(M mod x)*(Y mod x) = (M mod x)*(M mod x)*(Y mod x)*(Y mod x) = ((M mod x)^2) *((Y mod x)^2) = (M_0 mod x)^2. We interpret this to mean that after two months of combined practice of meditation and yoga there will be a shift into the vision-logic level of consciousness, and we therefore have the equation ((M mod x)*(Y mod x))^2 = I. We thus have a commutative biquasi 2-group consisting of the following 8 elements: M mod x, (M mod x)^2, (M mod x)^3, Y mod x, (M mod x)*(Y mod x), ((M mod x)^2)*(y mod x), ((M mod x)^3)*(Y mod x), and I. We note that our commutative assumption implies that ((M mod x)^2)*(Y mod x) = (M mod x)*((M mod x)*(Y mod x) = (M mod x)*(Ymod x)*(M mod x) = (Y mod x) * (M mod x)*(M mod x) = (Y mod x)*((M mod x)^2), and similarly ((M mod x)^3)*(Y mod x) = (Y mod x)*((M mod x)^3). Indeed we can even interpret this biquasi 2-group to be a cyclic biquasi-group of four elements, generated by M mod x, if we assume from the equations (M mod x)^4 = I and (Y mod x)^2 = I that Y mod x = (M mod x)^2. However, we may like to assume there are enough qualitative differences between our meditation and yoga practice that even though it does take exactly twice as long to shift into the vision-logic level of consciousness thru meditation alone compared to yoga alone, we do not want to reduce this to the cyclic biquasi-group structure of Y mod x = (M mod x)^2. In other words, we may want to assume that our original non-cyclic biquasi-group structure of 8 elements is a more accurate mathematical representation of our situation.

To be even more realistic, let us now assume that it does make a difference whether we first meditate or do yoga; i.e. (M mod x)*(Y mod x) is not equal to (Y mod x)*(M mod x), meaning that our biquasi-group is no longer commutative. According to group theory, there are exactly four possible groups consisting of 8 elements; the cyclic group which can be described in general by a^8 = I, a non-cyclic commutative group which can be described by a^4 = I, b^2 = I, which is precisely our above non-cyclic combined meditation and yoga group, and two groups that are not commutative. These two groups are known as the "dihedral" group, described by a^4 = I, b^2 = I, a*b = b*a^(-1), and the "quaternion" group, described by a^4 = I, b^4 = I, a^2 = b^2, a*b = b*a^(-1). However, if we try to generalize the dihedral group and quaternion group to our meditation/yoga biquasi-group situation, it no longer makes sense in terms of what is actually taking place. To see this, assume that we have the dihedral group formulation (M modx)^4 = I, (Y mod x)^2 = I, and (M mod x)*(Y mod x) = (Y mod x)*(M mod x)^(-1). If we multiply this last equation by (M mod x)^(-1) on the left and by M mod x on the right, we obtain ((M mod x)^(-1))*(M mod x)***(Y mod x)***(M mod x) = ((M mod x)^(-1))*(Y mod x)* ((M mod x)^(-1))*(M mod x) or (Y_0 mod x)*(M_0 mod x) = ((M_0 mod x)^(-1))*(Y_0 mod x). Our original equation means that doing a month of meditation followed by a month of yoga is equivalent to first doing a month of yoga followed by 3 months of meditation, since (Mod x)^4 = I implies that (Mod x)^(-1) = (Mod x)^3, implying that it is more beneficial to meditate first. However, our derived equation reverses the situation in the vision-logic level of consciousness, implying that doing a month of yoga followed by a month of meditation is equivalent to first doing 3 months meditation followed by a month of yoga, implying that it is more beneficial to do yoga first. Clearly, although our non-commutative equations may be legitimate mathematically, it would be difficult for us to resolve the apparent contradiction when we apply it to our biquasi-group meditation/yoga example, and we therefore reject the dihedral group as a legitimate model for our situation (the quaternion group would be rejected as well for the same reason). We thus find that our mathematical biquasi-group analysis does not allow for the more subtle aspects of the bona-fide situation that may arise, such as what is the impact of changing the order of whether we begin our practice by doing meditation or yoga. Our mathematical model may be useful up to a point, but it is crucial to not over-interpret the usefulness and accuracy of this mathematical model describing the inner realities of what is truly going on when a person shifts into higher levels of consciousness.

In conclusion, keeping the limitations of our mathematical model in mind, we see a glimpse of how mathematical group theory can be an interesting and useful device to begin a quantitative analysis of shifting into higher levels of consciousness in Ken Wilber's Integral theory. The mathematical complications quickly arise as we introduce various persons, in combination with various levels, streams, and states of consciousness, in various degrees of impact. However, the basic idea that we have described here is that mathematical group theory enables one to begin a comparative mathematical analysis in a structured ordered logical format to help us understand shifts into higher levels of consciousness. In actuality we can use our group theoretical model in a similar manner to describe the reverse condition; i.e.

how certain particular experiences for particular people can have significant impact toward a person shifting into a lower level of consciousness. But in regard to the specific examples that I have given to explain my mathematical model, the group theoretical model that I have proposed to describe shifts into higher levels of consciousness can be empirically tested through studying individuals engaged in a structured form of meditation over a certain period of time, as was done in the study conducted in the 1980s by Daniel P. Brown and Jack Engler, as described in the book "Transformations Of Consciousness" edited by Wilber, Engler, and Brown. A number of particular higher levels of consciousness were specifically defined in this study, and shifts into higher levels of consciousness were described, with the Rorschach test utilized as the measuring instrument. For a given individual who progressed into a higher level of consciousness in this study, I believe that my group theoretical mathematical model would accurately describe this consciousness level shift in accordance with the amount of time it took this individual to make the shift. My model would also mathematically reflect the differences in time for consciousness level shifts for different individuals, as well as the fact that some individuals were not able to shift into higher levels of consciousness no matter how long they meditated for. To extend the empirical testing possibilities, different types of meditative activity (including different types of meditation as well as related activities such as yoga, Tai Chi, Reiki, etc.) could be described thru this model, both for the same individual experiencing these different forms of meditative activity as well as for different individuals. Although there is an obvious advantage of having a controlled environment to undertake such a study, my mathematical model could be applied to more natural settings as well, assuming that there is a structured disciplined meditative activity timeframe being followed. Of-course an agreed upon relatively accurate measuring device to describe shifts into higher levels of consciousness must first be established, whether it be the Rorschach test or some other measuring device, before any kind of mathematical analysis can be undertaken. I believe that the biggest challenge in empirically testing a mathematical model such as the one I have described is actually with the measurement of these consciousness level shifts rather than with the mathematical model itself. For once something is measured and categorized it is quite reasonable to put a mathematical structure upon it; however, the question will always remain as to how accurately the measuring device truly reflects the deeper intrinsic mode of consciousness being measured. Of-course with our mathematical structure we are operating completely on a rational/scientific mode of inquiry here, i.e. we are completely in the orange meme in regard to Graves, Cowan, and Beck's Spiral Dynamics theory. But hopefully we are undertaking this orange analysis from a higher second tier level of thinking context, so all is well and we can thoroughly enjoy our preliminary mathematical group theoretical exploration of shifting into higher levels of consciousness in Ken Wilber's Integral theory.

APPENDIX: THE MATHEMATICAL THEORY OF BIQUASI-GROUPS

We now give more formal mathematical definitions and prove some basic lemmas to establish the mathematical properties of the biquasi-groups that we have introduced in relation to describing shifts into higher levels of consciousness.

Definition of Biquasi-Group: Let S and S_0 be sets and f a one-to-one and onto mapping of S such that f(S) = S_0 and if x is in S then f(x) = x_0 is in S_0; we refer to S_0 as the "transformed set" of S. We form a biquasi-group structure G on S and S_0 with operation *, and say that S is a "biquasi-group", if the following four properties hold:

1) if x is in S U S_0 and y is in S U S_0, then x*y is in S U S_0 (S U S_0 is defined to be the "union" of S and S_0, i.e. all elements that are either in S or S_0).
2) for all x, y, z in S U S_0, x*(y*z) = (x*y)*z (associative law).
3) there exists a "biquasi-identity element" I for S such that for all x in S, I*x = x* I = x_0, and I*I = I_0 = I.
4) for each x in S U S_0, there exists a "biquasi-inverse element" x^(-1) in S U S_0 such that x*x^(-1) = (x^(-1))*x = I (the symbol ^ denotes that the next number or letter is an exponent).

Given a non-negative integer n, if there are exactly n elements in a biquasi-group S we will say that S has order n; otherwise we will say that S has infinite order.

If for all x in S U S_0 and y in S U S_0 we have x*y = y*x, then we will say that our biquasi-group is "commutative".

If there exists an element x in S U S_0 such that for every element y, not equal to 1, in S U S_0 we have y = x^n for some positive integer n, where x^(-n) = (x^(-1))^n and x^0 = I, then we will say that our biquasi-group S is "cyclic" with generator x.

Given a biquasi-group S with transformed set S_0 and operation *, and a second biquasi-group T with transformed set T_0 and operation X, we will say that S and T are "isomorphic" if there exists a one-to-one and onto mapping f from S U S_0 onto T U T_0 with f(S) = T, f(S_0 = T_0, such that for all elements x and y in S U S_0 we have f(x*y) = f(x) X f(y), and such that for any x_0 in S_0 and (f(x))_0 in T_0 we have f(x_0) = (f(x))_0.

Lemma 1: If a biquasi-group S is cyclic, then S is commutative.

Proof: Since S is cyclic there exists a generator element g in S U S_0 such that for any elements x and y in S we have x*y = (g^m)*(g^n) = g^(m+n) for some integers m and n. However, we also have g^(m+n) = (g^n)*(g^m) = y*x and we therefore see that S is commutative.

Lemma 2: If x_0 is in S_0, then (x_0) *I = I*x_0 = x_0.

Proof If x_0 is in S_0 then x_0 = x*I = I*x for an element x in S. We therefore have (x_0)*I = (x*I)*I = x*(I*I) = x*I = x_0. Similarly, I*x_0 = I*(I*x) = (I*I)*x = I*x = x_0.:

Lemma 3: If x is in S_0 and y is in S U S_0, then x*y is in S_0.

Proof: If y is in S_0, then there exists an element y _1 in S such that y = I*y_1; similarly there exists an element x_1 in S such that x = (x_1)*I. We therefore have x*y = ((x_1)*I)*(I*y_1) = (x_1)*(I*I)*y_1 = ((x_1)*I)*y_1 = (I*x_1)*y_1 = I*((x_1)*(y_1)) and it therefore follows that x*y is in S_0.
If y is in S then x*y = ((x_1)*I)*y = (I*x_1)*y = I*((x_1)*y) and it once again follows that x*y is in S_0.

Lemma 4: If x is in S U S_0 and x*y = x*z or y*x = y*z, for y and z in S U S_0, then either y = z or y = z_0.

Proof: If x*y = x*z then (x^-1)*(x*y) = (x^-1)*(x*z); therefore ((x^-1)*x)*y = ((x^-1)*x)*z and I*y = I*z. It follows that either y = z or y = z_0. If y*x = z*x = I then by the analogous procedure we see that (y*x)*x^-1 = (z*x)*x^-1 and y*I = z*I; it once again follows that either y = z or y = z_0.

Lemma 5: In a biquasi-group S, the biquasi-identity element I is unique.

Proof: Assume that there exist two biquasi-identity elements I and J such that for all x in S, x*I = I*x = x_0, and x*J = J*x = x_0; we therefore have x*I = x*J. By Lemma 4 we know that either I = J or I = J_0. Since J is a biquasi-identity element for S, we have J_0 = J*J = J and therefore we can conclude that I = J and the biquasi-identity element is unique.

Lemma 6: Let S and T be isomorphic biquasi-groups such that f(S) = T and f(S_0) = T_0, where f is the one-to-one and onto mapping which satisfies the isomorphic biquasi-group requirements. Then
A) f(I) is the biquasi-identity element for T
B) for all x in S U S_0, f(x^-1) = (f(x))^-1.

Proof: Since f is an onto mapping, every element in T U T_0 has the form f(x) for some x in S U S_0
For all x in S U S_0 we have f(x) X f(I) = f(x*I) = f(x_0) = (f(x))_0. Similarly we have f(I) x f(x) = f(I*x) = f(x_0) = (f(x))_0 and therefore f(I) is the biquasi-identity element for T, which proves statement A. To prove statement B, we observe that for all x in S U S_0 we have f(x) X f(x^(-1)) = f(x*x^(-1)) = f(I) and f(x^(-1)) X f(x) = f((x^(-1)*x) = f(I). Since f(I) is the biquasi-identity element for T we see that f(x^(-1)) is a biquasi-inverse element of f(x); i.e. f(x^(-1)) = (f(x))^(-1).